



EPQ model for deteriorating items with carbon emission having multivariate demand considering freshness under shortages

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Abstract: There is a growing health consciousness among consumers and they are paying attention to the issue of greenhouse gas emissions, one of the major environmental threats. That's why companies are more and more eager to use green components. Currently, technological advances are causing markets to become more volatile and consumer preferences to change more rapidly. Thus, a product's life cycle is shortening every day, affecting its demolition rate. There are several factors which can influence its demand, including the price, on-hand inventory, and the freshness of the product. On the other hand, perishable items are prone to causing damage to the atmosphere if not handled properly. Therefore, the handling of deteriorated items must be done in an effective and efficient manner to prevent environmental hazards. The purpose of this research is therefore to develop a model for inventory management involving perishable items that is constrained both by physical degradations as well as freshness degradations. Price, stock availability, and the life cycle of products all affect the demand for the items. In this model we have used the linear price, time, and stock and freshness dependent demand. Shortage is taken into consideration, which is fully backlogged. Carbon emission has been considered to be minimized by using the carbon tax.

Keywords: EPQ model, carbon emission, multivariate demand, shortages

1. Introduction



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Organizations around the world rely on inventory management for success. By controlling materials from the point of purchase until the sales decision maker's perspective, overstocks and stockouts can be avoided. It is stated by Yavari et al. (2020), a significant challenge that companies are faced with during the process of managing their inventories cannot be ignored when looking at the intrinsic deterioration during the time that most items are kept, resulting in a decrease in both the quality and the freshness of the item as it continues to age, making it impossible for the item to remain in the market after completing its life cycle. Tirkolaei et al. (2017) reported that this problem commonly manifests itself in food products (for example, dairy products, vegetables, and meat).

Furthermore, in the current economic climate, consumers are more concerned about their own health when purchasing fresh items. Accordingly, it is common for consumers to prefer to purchase goods before they reach the end of their life cycle. It is for the purpose of storing them for a longer period of time. Therefore, businesses must cautiously handle and control the number of fresh items in their inventory stored in a warehouse. Some researchers have developed their model along the same line of research, such as Tomlin (2009), Herbon et al. (2012), Wahjudi et al. (2018), and Sun et al. (2019) with different assumptions.

Agi and Soni (2020) are proposing a mathematical model of inventory for the purpose of joint estimation of the optimal pricing and the storage of inventory that will allow surplus inventory to be accumulated at the end of the production cycle. Several authors have suggested that concluding the replenishment period on a positive note would be beneficial for all concerned. It is not desirable to maintain an inventory of deteriorated items at the end of their lifecycle, since they are unable to be stored for the next inventory cycle. Therefore, it is necessary to sell them at a salvage price in order to cover their costs.

Contributing to the improvement of society and the environment is of the utmost importance in the modern world. Therefore, the products' life cycle has been shortened, resulting in an increase in the number of products that are discarded. In the fashion industry, this cycle seems to be much shorter, as it appears to go out of fashion much more rapidly than most other industries. As time passes, the number of years a product has a lifespan is shrinking, or simply put, a product that had a lifespan of twenty years or more some decades ago now has a lifespan of half or less. Due to these sudden changes, however, one can define the new development of products, where the companies are utilizing the hardware of previous products, thereby

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reducing the burden on our resources. Moreover, it is economically beneficial for the companies as well. The green supply chain is an essential tool in every business to maximize profit with a low burden on the environment and providing finished goods to the market as per the demand. A short-life cycle product design with uncertain delivery time and demand has been developed by Weng and McClurg (2003). In the study by Reiner et al. According to a study (2009), if the effect of the product lifecycle is not taken into account while estimating demand, then the estimation is complicated, and there is a high likelihood of dumping inventory. In addition, Wee and Chang (2009) have developed a product life cycle that takes into account remanufacturing as well as deterioration.

Darwish (2014) formulated a classical stochastic continuous inventory control model. In this model, the production rate is finite and unmet demand is partially backordered. He also considered partially backlogging shortages. Mishra et al. (2020) studied a sustainable economic production quantity (SEPQ) model. This model has been compared in two parts, in which the first part saw the carbon tax and cap with shortage, and the second part looked at without shortage. Green technology (GT) and control of carbon emissions have also been considered.

Sarkar B. et al.(2016) described an integrated inventory model in which the Stackelberg game strategy is to optimize the overall cost of a vendor and buyer system together. After the lot is received, the buyer initiates an investigation to determine the defective items. After receiving the next lot, the buyer gives the vendor all the item's worth. This model has seen the dynamism of carbon emissions which is affected at some cost. Manna et al. (2020) examined an incomplete production inventory model with random carbon emissions under constant prepayment. An advance payment policy has been adopted in this paper, based on which the transportation cost has been kept free. Carbon emissions have been added to the decision to produce and transport from manufacturer to seller. Paul et al. (2022) considered an inventory model with retail investment in green operations with varying holding costs. In the given model, the demand rate depends on the selling price and the green technical session. Here, the effect of greenery on the cost and selling prices is shown. Sarkar et al. (2022) considered that green product innovation is viewed as more relevant than traditional non-

green products, mainly because of the rapid advancement of technology and low acceptance rate among consumers. Remanufacturing is analyzed in this model to look at the inherent risks involved and the three methods for innovation. Sarkar and Bhunia (2022) developed a flexible manufacturing-remanufacturing model based on green investment and improved service under variable demand. Singh, *et al.* (2022) developed inventory model for price-sensitive demand. They considers low carbon emission and partial backlogging.

2. Notation and assumptions

2.1 Notation

As part of the development of the inventory model, the following notation was used:

Parameters	Description
a	Parameter defining the scale of a price-dependent portion of the market demand.
b	Sensitivity parameter of a price-dependent portion of the market demand.
l	cycle length of the product
η	Salvage coefficient ($0 \leq \eta \leq 1$).
α	Sensitivity parameter based on the level of stock at present.
c_d	Deterioration cost
c	Purchase cost
K	Ordering cost
s	Salvage cost of deteriorated item

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h	Holding cost
h_c	Carbon mission cost due to the storage of the items
θ	Inventory deterioration rate ($0 \leq \theta \leq 1$).
d_c	Carbon mission cost due to the deterioration of the items
S_c	shortage cost

Decision variables

p	Selling price.
T	Replenishment period.

2.2 Assumptions

The production inventory model is developed here based on the following assumptions:

1. Deterioration is taken into consideration, which is constant in nature. The deteriorated items are salvaged at the end of the period.
2. The item has a short life cycle and after that it is not being suitable for the sale.
3. Shortages are allowed during the unavailability of the items which can be further backlogged.
4. Multivariate demand is considered here depending on the time, inventory level, product life cycle and price as follows

$$D(p, l, I(t), t) = \left(1 - \frac{t}{l}\right)(a - bp) + \alpha I(t)$$

where $l > 0, \alpha \geq 0, t \leq l$, and $0 \leq p \leq \frac{a}{b}$

3. Mathematical modeling

In Figure 1, we can see how the inventory level has changed over the course of time. As soon as the retailer obtains the order quantity at the beginning of the stocking cycle $t = 0$, inventory levels immediately begin to drop as both demand and deterioration cause the level of inventory to decrease, and it will reach zero units at the time t_1 . After that the shortages occur due to the absence of the items to the time $t = T$.

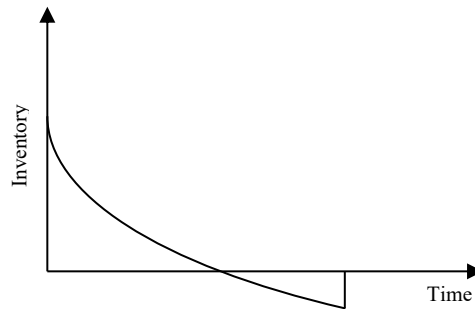


Figure 1. Display of inventory levels over time

On the basis of the assumptions outlined above, the following differential equation can be used to model the situation of the stock level:

$$\frac{dI(t)}{dt} + \theta I(t) = -\left\{\left(1 - \frac{t}{l}\right)(a - bp) + \alpha I(t)\right\} \quad I(t_1) = 0 \quad 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dI(t)}{dt} = -(a - bp) \quad I(t_1) = 0 \quad t_1 \leq t \leq T \quad (2)$$

The solution of the above equation can be expressed as below:

$$I(t) = \frac{(a - bp)}{(\alpha + \theta)} \left\{ \left(1 + \frac{1}{l(\alpha + \theta)} \right) (e^{(\alpha + \theta)(t_1 - t)} - 1) - \frac{1}{l} (t_1 e^{(\alpha + \theta)(t_1 - t)} - t) \right\} \quad 0 \leq t \leq t_1 \quad (3)$$

$$I(t) = (a - bp)(t_1 - t) \quad t_1 \leq t \leq T \quad (4)$$

The maximum inventory level q , which occurs at time zero can be expressed as follows:

$$q = \frac{(a - bp)}{(\alpha + \theta)} \left\{ - \left(1 + \frac{1}{l(\alpha + \theta)} \right) + \frac{e^{(\alpha + \theta)t_1}}{l} \left(l - t_1 + \frac{1}{(\alpha + \theta)} \right) \right\} \quad (5)$$

Maximum shortage

$$S = -I(T) = (a - bp)(T - t_1) \quad (6)$$

Order quantity

$$Q = q + S = \frac{e^{(\alpha + \theta)t_1}}{(\alpha + \theta)} \left(-1 - \frac{1}{l(\alpha + \theta)} \right) + \frac{(a - bp)e^{(\alpha + \theta)t_1}}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) + (a - bp)(T - t_1) \quad (7)$$

A system's profit is determined by the difference between its revenue and its total costs. The following section provides a detailed explanation of each component of the profit function.

1. Sales revenue

2. The supplier receives the revenue by selling the products as follows

$$SR = p \left\{ \int_0^{t_1} \left((a - bp) \left(1 - \frac{t}{l} \right) + \alpha I(t) \right) dt + \int_{t_1}^T (a - bp) dt \right\}$$

$$= p \left\{ (a - bp)t_1 \left(1 - \frac{t_1}{2l} \right) - \frac{\alpha(a - bp)t_1}{(\alpha + \theta)} \left(1 - \frac{t_1}{2l} + \frac{1}{l(\alpha + \theta)} \right) + \frac{\alpha(a - bp)}{(\alpha + \theta)} \frac{(e^{(\alpha + \theta)t_1} - 1)}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) + (a - bp)(T - t_1) \right\} \quad (8)$$

2. Salvage cost

The non-serviceable products are salvaged at a cost calculated as below $SV = s\eta \left\{ q - \int_0^{t_1} \left((a - bp) \left(1 - \frac{t}{l} \right) + \alpha I(t) \right) dt \right\}$

$$\begin{aligned}
 &= s\eta \left[-\frac{(a - bp)}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} \right) + \frac{(a - bp)e^{(\alpha + \theta)t_1}}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \right. \\
 &\quad \left. - \left\{ (a - bp)t_1 \left(1 - \frac{t_1}{2l} \right) - \frac{\alpha(a - bp)t_1}{(\alpha + \theta)} \left(1 - \frac{t_1}{2l} + \frac{1}{l(\alpha + \theta)} \right) \right. \right. \\
 &\quad \left. \left. + \left(\frac{e^{(\alpha + \theta)t_1} - 1}{(\alpha + \theta)} \right) \left(\frac{\alpha(a - bp)}{(\alpha + \theta)} \right) \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \right\} \right] \quad (9)
 \end{aligned}$$

3. Ordering cost

The cost of ordering the item is taken as $OC = K$ (10)

4. Holding cost

The items are stored for the time period $[0, t_1]$ and the cost carrying the inventory can be calculated as below

$$\begin{aligned}
 HC &= \int_0^{t_1} (h + h_c) I(t) dt \\
 &= (h + h_c) \left\{ \frac{(a - bp)t_1}{(\alpha + \theta)} \left[-1 + \frac{t_1}{2l} - \frac{1}{l(\alpha + \theta)} \right] + \left[\frac{e^{(\alpha + \theta)t_1} - 1}{(\alpha + \theta)} \right] \left[\frac{(a - bp)}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \right] \right\} \quad (11)
 \end{aligned}$$

5. The purchase cost

The finished products are purchased to sell further at the cost of

$$PC = cQ = c \left\{ \frac{(a - bp)}{(\alpha + \theta)} \left(-1 - \frac{1}{l(\alpha + \theta)} \right) + \frac{(a - bp)e^{(\alpha + \theta)t_1}}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) + (a - bp)(T - t_1) \right\} \quad (12)$$

6. The deterioration cost along

The items are deteriorate at the rate of θ and the opportunity cost lost due to the deteriorated items can be calculated as

$$\begin{aligned}
 DC &= (c_d + d_c) \left\{ q - \int_0^{t_1} \left((a - bp) \left(1 - \frac{t}{l} \right) + \alpha I(t) \right) dt \right\} \\
 &= (c_d + d_c) \left[-\frac{(a - bp)}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} \right) + \frac{(a - bp)e^{(\alpha + \theta)t_1}}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \right. \\
 &\quad \left. - \left\{ (a - bp)t_1 \left(1 - \frac{t_1}{2l} \right) - \frac{\alpha(a - bp)t_1}{(\alpha + \theta)} \left(1 - \frac{t_1}{2l} + \frac{1}{l(\alpha + \theta)} \right) \right\} \right. \\
 &\quad \left. + \frac{e^{(\alpha + \theta)t_1} - 1}{(\alpha + \theta)} \left(\frac{\alpha(a - bp)}{(\alpha + \theta)} \right) \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \right] \quad (13)
 \end{aligned}$$

7. The shortage cost

Stock out is unavoidable in certain conditions and hence we have assumed that the shortages are allowed which are further backlogged. Hence the shortage

$$\begin{aligned}
 SC &= \int_{t_1}^T S_c I(t) dt \\
 &= S_c (a - bp) \frac{(T - t_1)^2}{2}
 \end{aligned}$$

Therefore, the total profit per cycle is:

$$\begin{aligned}
 \pi(p, T) &= \frac{1}{T} \{ \text{sales revenue} + \text{salvage value} - \text{ordering cost} - \text{holding cost} - \text{carbon emission cost} \\
 &\quad - \text{purchase cost} - \text{shortage cost} - \text{deterioration cost} \}
 \end{aligned}$$

Hence, the total profit per unit of time is expressed as follows:

$$\begin{aligned}
 \pi(p, T) = & \frac{1}{T} \left[p \left\{ (a - bp)t_1 \left(1 - \frac{t_1}{2l} \right) - \frac{\alpha(a - bp)t_1}{(\alpha + \theta)} \left(1 - \frac{t_1}{2l} + \frac{1}{l(\alpha + \theta)} \right) + (a - bp)(T - t_1) \right. \right. \\
 & + \frac{\alpha(a - bp)}{(\alpha + \theta)} \frac{(e^{(\alpha + \theta)t_1} - 1)}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \left. \right\} + \eta \left[- \frac{(a - bp)}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} \right) \right. \\
 & + \frac{(a - bp)e^{(\alpha + \theta)t_1}}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) - \left\{ (a - bp)t_1 \left(1 - \frac{t_1}{2l} \right) - \frac{\alpha(a - bp)t_1}{(\alpha + \theta)} \right. \\
 & \left. \left(1 - \frac{t_1}{2l} + \frac{1}{l(\alpha + \theta)} \right) + \frac{e^{(\alpha + \theta)t_1} - 1}{(\alpha + \theta)} \left(\frac{\alpha(a - bp)}{(\alpha + \theta)} \right) \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \right\} \left. \right] \\
 & - \left\{ (h + h_c) \left\{ \frac{(a - bp)t_1}{(\alpha + \theta)} \left(-1 + \frac{t_1}{2l} - \frac{1}{l(\alpha + \theta)} \right) + \left(\frac{e^{(\alpha + \theta)t_1} - 1}{(\alpha + \theta)} \right) \left(\frac{(a - bp)}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \right) \right\} \right. \\
 & + c \left\{ \frac{(a - bp)}{(\alpha + \theta)} \left(-1 - \frac{1}{l(\alpha + \theta)} \right) + \frac{(a - bp)e^{(\alpha + \theta)t_1}}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) + (a - bp)(T - t_1) \right\} \\
 & + (c_d + d_c) \left[- \frac{(a - bp)}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} \right) + \frac{(a - bp)e^{(\alpha + \theta)t_1}}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \right. \\
 & - \left\{ (a - bp)t_1 \left(1 - \frac{t_1}{2l} \right) - \frac{\alpha(a - bp)t_1}{(\alpha + \theta)} \left(1 - \frac{t_1}{2l} + \frac{1}{l(\alpha + \theta)} \right) \right. \\
 & \left. \left. + \frac{e^{(\alpha + \theta)t_1} - 1}{(\alpha + \theta)} \left(\frac{\alpha(a - bp)}{(\alpha + \theta)} \right) \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \right\} \right] + K + S_c(a - bp) \frac{(T - t_1)^2}{2} \left. \right] \quad (15)
 \end{aligned}$$

Equation (15) gives an expression of the total profit per unit of time, and it can be shown that this result is a function of the selling price (p) as well as the replenishment cycle time (T). In the following section, we describe the solution procedure.

4. Solution procedure

It is necessary to meet the following conditions in order to maximize the total profit $\pi(p, T)$ per unit of time function as follows:

$$\frac{\partial \pi(p, T)}{\partial p} = 0 \quad (16)$$

and

$$\frac{\partial \pi(p, T)}{\partial T} = 0 \quad (17)$$

Moreover, in order to ensure that the expected total profit per unit of time will be concave, the following conditions must be met:

$$\frac{\partial^2 \pi(p, T)}{\partial p^2} < 0 \quad (18)$$

$$\frac{\partial^2 \pi(p, T)}{\partial T^2} < 0 \quad (19)$$

$$\frac{\partial^2 \pi(p, T)}{\partial p \partial T} = \frac{\partial^2 \pi(p, T)}{\partial T \partial p} \quad (20)$$

$$\left(\frac{\partial^2 \pi(p, T)}{\partial p^2} \right) \left(\frac{\partial^2 \pi(p, T)}{\partial T^2} \right) - \left(\frac{\partial^2 \pi(p, T)}{\partial p \partial T} \right) \left(\frac{\partial^2 \pi(p, T)}{\partial T \partial p} \right) > 0 \quad (21)$$

The first partial derivative of $\pi(p, T)$ with respect to p can be represented as:

$$\begin{aligned} \frac{\partial \pi(p, T)}{\partial p} = & \frac{1}{T} \left[(a - 2bp) \left\{ t_1 \left(1 - \frac{t_1}{2l} \right) - \frac{\alpha t_1}{(\alpha + \theta)} \left(1 - \frac{t_1}{2l} + \frac{1}{l(\alpha + \theta)} \right) + (T - t_1) \right. \right. \\ & \left. \left. + \frac{\alpha}{(\alpha + \theta)} \frac{(e^{(\alpha + \theta)t_1} - 1)}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \right\} + \eta(-b) \left[-\frac{1}{(\alpha + \theta)} \left(1 + \frac{1}{l(\alpha + \theta)} \right) \right] \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{e^{(\alpha+\theta)t_1}}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} - \frac{t_1}{l} \right) - \left\{ t_1 \left(1 - \frac{t_1}{2l} \right) - \frac{\alpha t_1}{(\alpha+\theta)} \right. \\
& \left. \left(1 - \frac{t_1}{2l} + \frac{1}{l(\alpha+\theta)} \right) + \frac{e^{(\alpha+\theta)t_1} - 1}{(\alpha+\theta)} \left(\frac{\alpha}{(\alpha+\theta)} \right) \left(1 + \frac{1}{l(\alpha+\theta)} - \frac{t_1}{l} \right) \right\} \\
& + b \left\{ (h + h_c) \left\{ \frac{t_1}{(\alpha+\theta)} \left(-1 + \frac{t_1}{2l} - \frac{1}{l(\alpha+\theta)} \right) + \left(\frac{e^{(\alpha+\theta)t_1} - 1}{(\alpha+\theta)} \right) \left(\frac{1}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} - \frac{t_1}{l} \right) \right) \right\} \right. \\
& \left. + c \left\{ \frac{1}{(\alpha+\theta)} \left(-1 - \frac{1}{l(\alpha+\theta)} \right) + \frac{e^{(\alpha+\theta)t_1}}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} - \frac{t_1}{l} \right) + (T - t_1) \right\} \right. \\
& \left. + (c_d + d_c) \left[-\frac{1}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} \right) + \frac{e^{(\alpha+\theta)t_1}}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} - \frac{t_1}{l} \right) \right] \right\} \quad (22)
\end{aligned}$$

While the first partial derivative of $\pi(p, T)$ with respect to T can be written as:

$$\begin{aligned}
\frac{\partial \pi(p, T)}{\partial T} = & -\frac{1}{T^2} \left[p \left\{ (a - bp)t_1 \left(1 - \frac{t_1}{2l} \right) - \frac{\alpha(a - bp)t_1}{(\alpha+\theta)} \left(1 - \frac{t_1}{2l} + \frac{1}{l(\alpha+\theta)} \right) + (a - bp)(-t_1) \right. \right. \\
& + \frac{\alpha(a - bp)}{(\alpha+\theta)} \frac{(e^{(\alpha+\theta)t_1} - 1)}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} - \frac{t_1}{l} \right) \left. \right\} + \eta \left[-\frac{(a - bp)}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} \right) \right. \\
& + \frac{(a - bp)e^{(\alpha+\theta)t_1}}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} - \frac{t_1}{l} \right) - \left\{ (a - bp)t_1 \left(1 - \frac{t_1}{2l} \right) - \frac{\alpha(a - bp)t_1}{(\alpha+\theta)} \right. \\
& \left. \left. \left(1 - \frac{t_1}{2l} + \frac{1}{l(\alpha+\theta)} \right) + \frac{e^{(\alpha+\theta)t_1} - 1}{(\alpha+\theta)} \left(\frac{\alpha(a - bp)}{(\alpha+\theta)} \right) \left(1 + \frac{1}{l(\alpha+\theta)} - \frac{t_1}{l} \right) \right\} \right] \\
& - \left\{ (h + h_c) \left\{ \frac{(a - bp)t_1}{(\alpha+\theta)} \left(-1 + \frac{t_1}{2l} - \frac{1}{l(\alpha+\theta)} \right) + \left(\frac{e^{(\alpha+\theta)t_1} - 1}{(\alpha+\theta)} \right) \left(\frac{(a - bp)}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} - \frac{t_1}{l} \right) \right) \right\} \right. \\
& + c \left\{ \frac{(a - bp)}{(\alpha+\theta)} \left(-1 - \frac{1}{l(\alpha+\theta)} \right) + \frac{(a - bp)e^{(\alpha+\theta)t_1}}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} - \frac{t_1}{l} \right) + (a - bp)(-t_1) \right\} \\
& + (c_d + d_c) \left[-\frac{(a - bp)}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} \right) + \frac{(a - bp)e^{(\alpha+\theta)t_1}}{(\alpha+\theta)} \left(1 + \frac{1}{l(\alpha+\theta)} - \frac{t_1}{l} \right) \right. \\
& \left. \left. - \left\{ (a - bp)t_1 \left(1 - \frac{t_1}{2l} \right) - \frac{\alpha(a - bp)t_1}{(\alpha+\theta)} \left(1 - \frac{t_1}{2l} + \frac{1}{l(\alpha+\theta)} \right) \right\} \right] \right]
\end{aligned}$$

$$+ \frac{e^{(\alpha+\theta)t_1} - 1}{(\alpha + \theta)} \left(\frac{\alpha(a - bp)}{(\alpha + \theta)} \right) \left(1 + \frac{1}{l(\alpha + \theta)} - \frac{t_1}{l} \right) \left. \right] + K + S_c(a - bp) \frac{t_1^2}{2} \left. \right] - \frac{S_c(a - bp)}{2} = 0$$

(23)

Example 1. Consider the following example to illustrate a real-life scenario. the parametric values for this model are taken as follows:

$$a = 600, b = 20, l = 1, K = 250, c = 5, s = 4, \alpha = 0.5, \theta = 0.05, h = 1.75, c_d = 2,$$

By using the above solution procedure, the following optimal solution for the inventory system are calculated:

$$T^* = 0.44, Q^* = 94.43, p^* = 17.69, \text{ and } \pi^*(p^*, T^*) = 2049.90.$$

The optimal results satisfy the necessary and sufficient conditions of optimality that is;

$$\frac{\partial^2 \pi(p, T)}{\partial p^2} = -34.56784 < 0, \quad \frac{\partial^2 \pi(p, T)}{\partial T^2} = -49710.46 < 0, \quad \frac{\partial^2 \pi(p, T)}{\partial p \partial T} = \frac{\partial^2 \pi(p, T)}{\partial T \partial p} = 13.01441,$$

$$\text{and } \left(\frac{\partial^2 \pi(p, T)}{\partial p^2} \right) \left(\frac{\partial^2 \pi(p, T)}{\partial T^2} \right) - \left(\frac{\partial^2 \pi(p, T)}{\partial p \partial T} \right) \left(\frac{\partial^2 \pi(p, T)}{\partial T \partial p} \right) = 0.1718214 \times 10^7 > 0.$$

In this case, at the total profit function, there will be a strict concavity as a result of the fact that the hessian determinant of the function is higher than zero, as well as the Hessian matrix is negative definite. As a result of this, we are able to come up with an optimal solution.

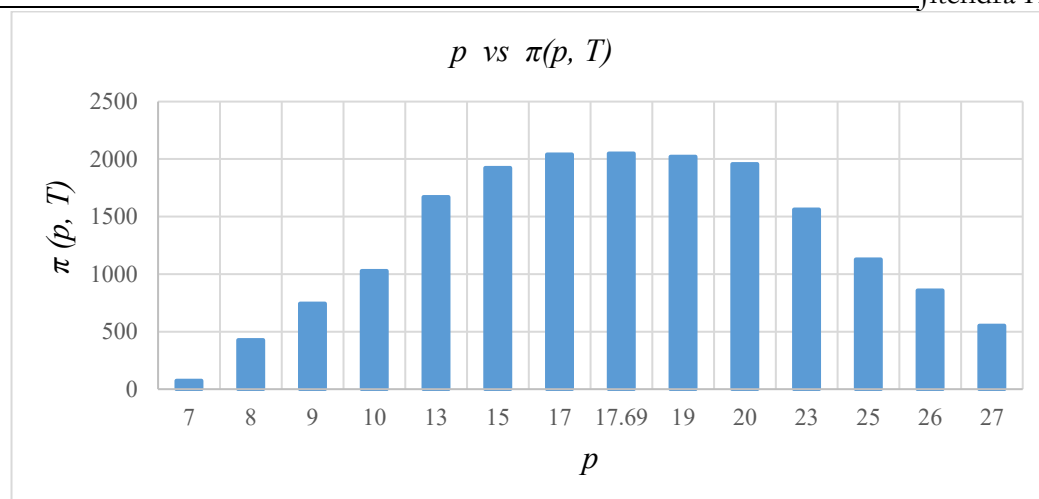


Figure 2. The behavior of the profit function $\pi(p, T)$ with respect to price.

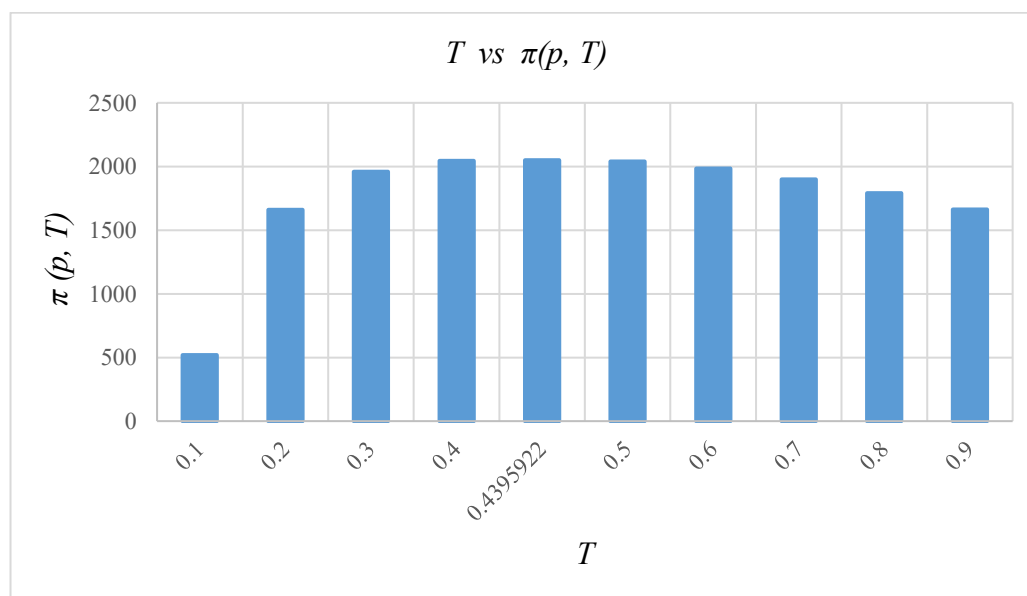


Figure 3. The behavior of the profit function $\pi(p, T)$ with respect to time.

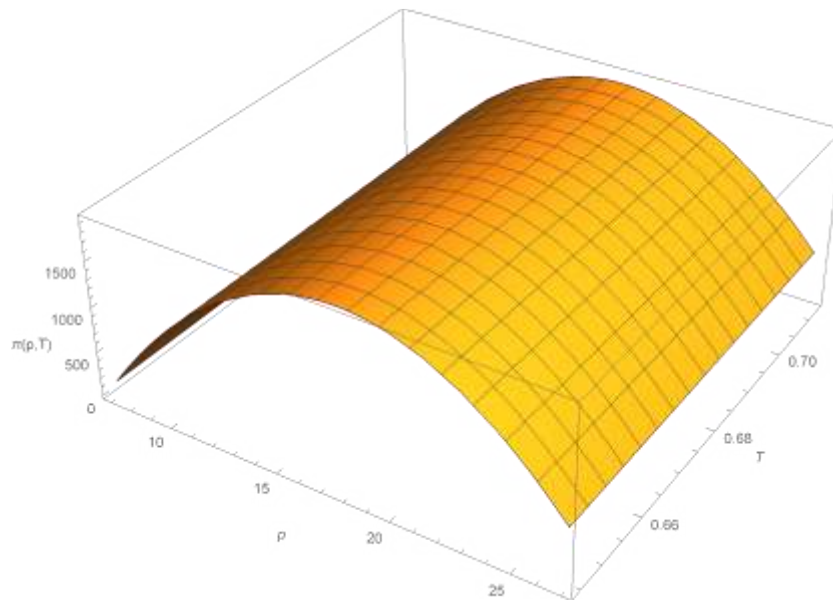


Figure 4. The concavity of the profit function $\pi(p, T)$ with respect to price and time

6. Sensitivity analysis

An analysis of the sensitivity of the mathematical model developed here is performed. We are using distinct values for each of the input parameters, while we are keeping all other input parameters unchanged. As can be seen in Table 1, the results of the study are summarized.

Table 1. Sensitivity analysis of the optimal results.

Parameter		T	p	Q	<i>Profit</i>
l	0.8	0.38	17.66	78.83	1874.13
	0.9	0.41	17.68	86.63	1968.36
	1	0.44	17.69	94.43	2049.90

	1.1	0.47	17.70	102.29	2121.73
	1.2	0.50	17.72	110.26	2185.93
K	250	0.44	17.69	94.43	2049.90
	300	0.48	17.70	100.68	1940.79
	350	0.51	17.72	106.04	1839.47
	400	0.54	17.73	110.72	1744.32
	450	0.57	17.74	114.83	1654.23
α	0.25	0.41	17.68	84.91	1920.09
	0.35	0.42	17.69	88.45	1970.20
	0.45	0.43	17.69	92.34	2022.69
	0.5	0.44	17.69	94.43	2049.90
	0.55	0.45	17.69	96.63	2077.81
θ	0.03	0.44	17.68	94.17	2052.52
	0.04	0.44	17.69	94.30	2051.21
	0.05	0.44	17.69	94.43	2049.90
	0.06	0.44	17.70	94.56	2048.59
	0.07	0.44	17.70	94.69	2047.28
c	4	0.43	17.18	95.76	2269.81
	4.5	0.43	17.44	95.11	2158.58
	5	0.44	17.69	94.43	2049.90
	5.5	0.45	17.95	93.74	1943.77
	6	0.45	18.20	93.02	1840.19
s	3.6	0.44	17.69	94.39	2049.24
	3.8	0.44	17.69	94.41	2049.57
	4	0.44	17.69	94.43	2049.90
	4.2	0.44	17.69	94.45	2050.23
	4.4	0.44	17.69	94.47	2050.57

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h	1.65	0.44	17.68	94.75	2054.73
	1.70	0.44	17.69	94.59	2052.31
	1.75	0.44	17.69	94.43	2049.90
	1.80	0.44	17.70	94.27	2047.50
	1.85	0.44	17.70	94.11	2045.09
c_d	1	0.44	17.69	94.55	2051.97
	1.5	0.44	17.69	94.49	2050.94
	2	0.44	17.69	94.43	2049.90
	2.5	0.44	17.69	94.37	2048.87
	3	0.44	17.70	94.31	2047.84
η	0.6	0.44	17.69	94.34	2048.25
	0.7	0.44	17.69	94.38	2049.08
	0.8	0.44	17.69	94.43	2049.90
	0.9	0.44	17.69	94.48	2050.73
	1	0.44	17.69	94.52	2051.56
a	360	0.77	11.79	69.97	231.06
	480	0.55	14.73	85.22	956.05
	600	0.44	17.69	94.43	2049.90
	720	0.37	20.66	100.82	3508.12
	840	0.31	23.64	105.58	5328.86
b	5	0.22	62.60	58.93	14329.26
	10	0.30	32.64	76.55	6005.92
	20	0.44	17.69	94.43	2049.90
	25	0.50	14.71	99.05	1313.23
	30	0.57	12.74	101.73	847.18

7. Observations

- It is the same effect that product life cycle l and ordering cost K have on the cost of inventory as well as the total profit for the price dependent demand function. Longer product life cycles result in higher selling prices, longer optimal cycle times, and larger orders, resulting in higher sales prices. Prices and inventory cycle times increase as the ordering cost increases, but the overall profit tends to decrease. Then, company management should start taking actions to bring down the ordering costs, in order to generate more profits for the company.
- It has been observed that higher values of the parameter which is associated with the sensitivity of demand to inventory levels, lead to an extended inventory cycle time. This results in a rise in prices, thereby increasing profits. A higher demand sensitivity to the stock level results in a larger order quantity. Due to the fact that the sensitivity parameter does not directly depend on the decision maker, it is suggested that this parameter be set at high values in this case. This would enable high profits. This means that managers are more interested in exhibiting larger quantities of stock on shelf space.
- With an increase in the purchase cost c , the price and inventory cycle time will increase slightly. Contrary to this, an increase in this cost results in a reduction in profit. In such a case, the recommendation would be to develop low-cost purchasing policies in order to increase profits regardless of the function of demand.

8. Conclusions

This study aims to develop a model for inventory management that incorporates the pricing, stocking levels, as well as the time-dependent demand factors. Deterioration as well as the loss of freshness of perishable products is both considered. When it comes to working with perishable products, not only the costs associated with deterioration but also the salvage value of these items is taken into account during the entire process cycle.

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By utilizing the solution procedure, one can determine the optimal values for the price, the cycle time for the inventory to be stocked, and the quantity of the item to be purchased based on the optimal solution that has been established. We present a numerical illustration of the model, provided with some numerical examples, followed by a sensitivity analysis for each individual input parameter associated with the model. Based on the patterns observed in the sensitivity analysis, it can be concluded that, with increasing ordering costs, purchasing costs, and shelf-life, an increase can be observed as a result of an increase in a number of factors such as the optimal price, optimal cycle length, ordering quantity, and overall profits. It is also worth noting that an increase in the value of shelf life results in an increase in price and profits. Moreover, as the ordering cost increases for all functions, such as the amount of goods ordered, the inventory cycle time and the quantity ordered all increase for all functions. However, profits tend to decrease. A higher purchasing cost also results in an increase in both the price and the time that it takes for the inventory to cycle due to an increase in purchasing costs. However, a decrease in the quantity ordered and a decline in total profits are the result of a higher purchasing cost. The current research work aims to extend and significantly contribute towards the state-of-the-art in the field of inventory management, particularly in the case of perishable goods that have price-stock-time-dependent demand patterns. It should be noted that this inventory model is not without its limitations, which can be used as an opportunity to explore a number of extension possibilities that may be explored in the future as well as the need for further research. Firstly, there is the possibility of building an inventory model that has essentially the same characteristics and demand pattern as the current inventory model, in addition to incorporating sustainability elements into the model, which will allow us to evaluate the impact of carbon tax and cap-and-trade mechanisms on the environment. Secondly, a model that allows shortages to occur both with full or partial backlogs can be considered. The third factor to consider is the fact that the freshness degradation of non-instantaneous items can also be incorporated into the model of inventory that is being proposed. There are also other components that can be investigated, such as promotional policies or advertising efforts that can also be considered.

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